미분적분학1

Worksheet #11

학번	
성명	

[연습문제 7.2]

1-49. 다음 적분을 계산하여라.

2. $\int \sin^6 x \cos^3 x\, dx$

(풀이)

$u=\sin x$라 하자. 그러면 $du=\cos x\, dx$이므로

$$\int \sin^6 x\cos^3 x\, dx = \int \sin^6 x\cos^2 x\cos x\, dx$$
$$= \int \sin^6 x(1-\sin^2 x)\cos x\, dx$$
$$= \int u^6(1-u^2)du = \int (u^6-u^8)du$$
$$= \frac{1}{7}u^7 - \frac{1}{9}u^9 + C = \frac{1}{7}\sin^7 x - \frac{1}{9}\sin^9 x + C$$

11. $\int_0^{\frac{\pi}{2}} \sin^2 x\cos^2 x\, dx$

(풀이) $\int_0^{\frac{\pi}{2}} \sin^2 x\cos^2 x\, dx = \int_0^{\frac{\pi}{2}} \frac{1-\cos 2x}{2}\times\frac{1+\cos 2x}{2}dx$

$$= \int_0^{\frac{\pi}{2}} \frac{1-\cos^2 2x}{4}dx$$
$$= \int_0^{\frac{\pi}{2}} \frac{1}{4}\left(1-\frac{1+\cos 4x}{2}\right)dx$$
$$= \int_0^{\frac{\pi}{2}} \left(\frac{1}{8}-\frac{1}{8}\cos 4x\right)dx$$
$$= \left[\frac{1}{8}x - \frac{1}{32}\sin 4x\right]_0^{\frac{\pi}{2}} = \frac{\pi}{16}$$

21. $\int \tan x\sec^3 x dx$

(풀이)

$u=\sec x$라 하자. 그러면 $du=\sec x\tan x\, dx$이므로

$$\int \tan x\sec^3 x\, dx = \int \sec^2 x\times\sec x\tan x\, dx$$
$$= \int u^2 du = \frac{1}{3}u^3 + C = \frac{1}{3}\sec^3 x + C$$

25. $\int \tan^4 x\sec^6 x\, dx$

(풀이)

$u=\tan x$라 하자. 그러면 $du=\sec^2 x\, dx$이므로

$$\int \tan^4 x\sec^6 x\, dx = \int \tan^4 x\sec^4 x\times\sec^2 x\, dx$$
$$= \int \tan^4 x(\tan^2 x+1)^2\times\sec^2 x\, dx$$
$$= \int u^4(u^2+1)^2 du = \int (u^8+2u^6+u^4)du$$
$$= \frac{1}{9}u^9 + \frac{2}{7}u^7 + \frac{1}{5}u^5 + C$$
$$= \frac{1}{9}\tan^9 x + \frac{2}{7}\tan^7 x + \frac{1}{5}\tan^5 x + C$$

41. $\int \sin 8x\cos 5x\, dx$

(풀이)

$$\int \sin 8x\cos 5x\, dx$$
$$= \int \frac{1}{2}(\sin(8x-5x)+\sin(8x+5x))dx$$
$$= \int \frac{1}{2}(\sin 3x + \sin 13x)dx$$
$$= \frac{1}{2}\left(-\frac{1}{3}\cos 3x - \frac{1}{13}\cos 13x\right) + C$$

[연습문제 7.3]

4-30. 다음 적분을 계산하여라.

9. $\int_2^3 \frac{dx}{(x^2-1)^{\frac{3}{2}}}$

(풀이)

$x=\sec\theta$라 치환하자. 그러면 $dx=\sec\theta\tan\theta\, d\theta$이고, $x=2$일 때, $\sec\theta=2$이므로 $\theta=\frac{\pi}{3}$, $x=3$일 때, $\sec\theta=3$이므로 $\theta=\sec^{-1}3$이다. 따라서,

$$\int_2^3 \frac{dx}{(x^2-1)^{\frac{3}{2}}} = \int_2^3 \frac{1}{\left(\sqrt{x^2-1}\right)^3}dx$$

$$=\int_{\frac{\pi}{3}}^{\sec^{-1}3}\frac{1}{\left(\sqrt{\sec^2\theta-1}\right)^3}\times\sec\theta\tan\theta\,d\theta$$

$$=\int_{\frac{\pi}{3}}^{\sec^{-1}3}\frac{1}{\tan^3\theta}\times\sec\theta\tan\theta\,d\theta$$

$$=\int_{\frac{\pi}{3}}^{\sec^{-1}3}\frac{\cos^3\theta}{\sin^3\theta}\times\frac{1}{\cos\theta}\times\frac{\sin\theta}{\cos\theta}\,d\theta$$

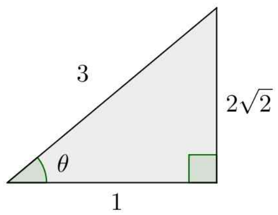
$$=\int_{\frac{\pi}{3}}^{\sec^{-1}3}\frac{1}{\sin\theta}\times\frac{\cos\theta}{\sin\theta}\,d\theta$$

$$=\int_{\frac{\pi}{3}}^{\sec^{-1}3}\csc\theta\cot\theta\,d\theta$$

$$=\left[-\csc\theta\right]_{\frac{\pi}{3}}^{\sec^{-1}3}$$

$$=-\csc(\sec^{-1}3)+\csc\frac{\pi}{3}$$

$$=-\frac{3}{2\sqrt{2}}+\frac{2}{\sqrt{3}}$$



여기서 $\theta=\sec^{-1}3$이라 하면 $\sec\theta=3$이므로 위의 삼각형에서 $\csc(\sec^{-1}3)=\csc\theta=\dfrac{3}{2\sqrt{2}}$이다.

10. $\displaystyle\int_0^{\frac{2}{3}}\sqrt{4-9x^2}\,dx$

(풀이)

$x=\dfrac{2}{3}\sin\theta$라 하자. 그러면 $dx=\dfrac{2}{3}\cos\theta\,d\theta$이고, $x=0$일 때, $0=\dfrac{2}{3}\sin\theta$이므로 $\theta=0$이고, $x=\dfrac{2}{3}$일 때, $\dfrac{2}{3}=\dfrac{2}{3}\sin\theta$이므로 $\theta=\dfrac{\pi}{2}$이다. 따라서,

$$\int_0^{\frac{2}{3}}\sqrt{4-9x^2}\,dx$$

$$=\int_0^{\frac{\pi}{2}}\sqrt{4-9\times\left(\frac{2}{3}\sin\theta\right)^2}\times\frac{2}{3}\cos\theta\,d\theta$$

$$=\int_0^{\frac{\pi}{2}}2\cos\theta\times\frac{2}{3}\cos\theta\,d\theta$$

$$=\int_0^{\frac{\pi}{2}}\frac{4}{3}\cos^2\theta\,d\theta$$

$$=\int_0^{\frac{\pi}{2}}\frac{4}{3}\times\frac{1+\cos2\theta}{2}\,d\theta$$

$$=\left[\frac{2}{3}\left(\theta+\frac{1}{2}\sin2\theta\right)\right]_0^{\frac{\pi}{2}}=\frac{\pi}{3}$$

14. $\displaystyle\int_0^1\frac{dx}{(x^2+1)^2}$

(풀이)

$x=\tan\theta$라 하자. 그러면 $dx=\sec^2\theta\,d\theta$이고, $x=0$일 때, $\tan\theta=0$이므로 $\theta=0$이고, $x=1$일 때, $\tan\theta=1$이므로 $\theta=\dfrac{\pi}{4}$이다. 따라서,

$$\int_0^1\frac{dx}{(x^2+1)^2}=\int_0^1\frac{1}{(x^2+1)^2}dx$$

$$=\int_0^{\frac{\pi}{4}}\frac{1}{(\tan^2\theta+1)^2}\times\sec^2\theta\,d\theta$$

$$=\int_0^{\frac{\pi}{4}}\frac{1}{\sec^4\theta}\times\sec^2\theta\,d\theta$$

$$=\int_0^{\frac{\pi}{4}}\frac{1}{\sec^2\theta}\,d\theta=\int_0^{\frac{\pi}{4}}\cos^2\theta\,d\theta$$

$$=\int_0^{\frac{\pi}{4}}\frac{1+\cos2\theta}{2}\,d\theta$$

$$=\left[\frac{1}{2}\left(\theta+\frac{1}{2}\sin2\theta\right)\right]_0^{\frac{\pi}{4}}$$

$$=\frac{1}{2}\left(\frac{\pi}{4}+\frac{1}{2}\sin\frac{\pi}{2}\right)=\frac{\pi}{8}+\frac{1}{4}$$

24. $\displaystyle\int_0^1\sqrt{x-x^2}\,dx$

(풀이)

$x-\dfrac{1}{2}=\dfrac{1}{2}\sin\theta$라 하자. 그러면 $dx=\dfrac{1}{2}\cos\theta\,d\theta$이고 $x=0$일 때, $\dfrac{1}{2}\sin\theta=-\dfrac{1}{2}$이므로 $\sin\theta=-1$에서 $\theta=-\dfrac{\pi}{2}$이고, $x=1$일 때, $\dfrac{1}{2}\sin\theta=\dfrac{1}{2}$이므로 $\sin\theta=1$에서 $\theta=\dfrac{\pi}{2}$이다. 따라서,

$$\int_0^1\sqrt{x-x^2}\,dx=\int_0^1\sqrt{\frac{1}{4}-\frac{1}{4}+x-x^2}\,dx$$

$$=\int_0^1\sqrt{\frac{1}{4}-\left(x-\frac{1}{2}\right)^2}\,dx$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\frac{1}{4} - \frac{1}{4}\sin^2\theta} \times \frac{1}{2}\cos\theta \, d\theta$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{4}\cos^2\theta \, d\theta = 2\int_0^{\frac{\pi}{2}} \frac{1}{4}\cos^2\theta \, d\theta$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{1+\cos 4\theta}{2} \, d\theta$$

$$= \frac{1}{4}\left[\theta + \frac{1}{4}\sin 4\theta\right]_0^{\frac{\pi}{2}} = \frac{\pi}{8}$$

27. $\int \sqrt{5+4x-x^2}\, dx$

(풀이)

$x-2 = 3\sin\theta$라 하자. 그러면 $dx = 3\cos\theta\, d\theta$이므로

$$\int \sqrt{5+4x-x^2}\, dx = \int \sqrt{9-4+4x-x^2}\, dx$$

$$= \int \sqrt{9-(x^2-4x+4)}\, dx$$

$$= \int \sqrt{9-(x-2)^2}\, dx$$

$$= \int \sqrt{9-9\sin^2\theta} \times 3\cos\theta \, d\theta$$

$$= \int 9\cos^2\theta\, d\theta = \int 9 \times \frac{1+\cos 2\theta}{2}\, d\theta$$

$$= \frac{9}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right) + C$$

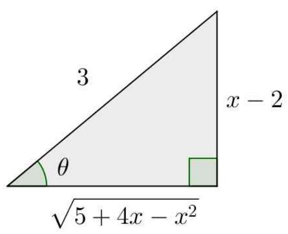
$$= \frac{9}{2}(\theta + \sin\theta\cos\theta) + C$$

$$= \frac{9}{2}\left(\sin^{-1}\left(\frac{x-2}{3}\right) + \frac{(x-2)\sqrt{5+4x-x^2}}{9}\right) + C$$

$$= \frac{9}{2}\sin^{-1}\left(\frac{x-2}{3}\right) + \frac{(x-2)\sqrt{5+4x-x^2}}{2} + C$$

여기서 $x-2 = 3\sin\theta$이므로 $\sin\theta = \frac{x-2}{3}$이고

$\theta = \sin^{-1}\left(\frac{x-2}{3}\right)$이다.



또한 위의 그림과 같으므로

$$\sin\theta\cos\theta = \frac{x-2}{3} \times \frac{\sqrt{5+4x-x^2}}{3}$$

$$= \frac{(x-2)\sqrt{5+4x-x^2}}{9}$$

이다.